A CLASS OF HIGHER ORDER ACCURATE SCHEMES FOR FLUID INTERFACE PROBLEMS

A THESIS

submitted by

HARI VANSH RAI MITTAL

for the award of the degree

of

DOCTOR OF PHILOSOPHY

SCHOOL OF BASIC SCIENCES

INDIAN INSTITUTE OF TECHNOLOGY MANDI

KAMAND-175005, INDIA

JULY, 2016

Dedicated

To

My Parents

THESIS CERTIFICATE

This is to certify that the thesis titled **"A CLASS OF HIGHER ORDER ACCURATE SCHEMES FOR FLUID INTERFACE PROBLEMS"**, submitted by **HARI VANSH RAI MITTAL**, to the Indian Institute of Technology Mandi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma.

SBS, IIT Mandi, Mandi

Email: rajendra@iitmandi.ac.in

Dr. Rajendra K. Ray

(Supervisor)

Declaration by the Research Scholar

I hereby declare that the entire work embodied in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of Dr. Rajendra K. Ray, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgments have been made wherever the work described is based on finding of other investigators.

Kamand 175005

Date:

Signature:

(HARI VANSH RAI MITTAL)

Acknowledgements

First and foremost, I offer my sincere gratitude to the almighty. Next, I am very grateful for the opportunity I have been given to do a PhD at the School of Basic Sciences at IIT Mandi. I would like to thank my supervisor Dr. Rajendra K. Ray who, with great enthusiasm and knowledge introduced me to the interesting and challenging field of Computational Fluid Dynamics. I appreciate all support and help during the work with this thesis and the way he has supervised me so that I could become an independent researcher.

I am also grateful to the other members of my doctoral committee: Dr. Syed Abbas, Dr. Manoj Thakur and Dr. P. Anil Kishan for their valuable suggestions and to my teachers who encouraged me to pursue higher studies since my childhood, Mr. Saji Pilley, Mr. Avtar Singh, Dr. Jatinder Grover, Dr. Yograj Angrish. I acknowledge Dr. Jiten C. Kalita from IIT Guwahati for the collaboration. His experience and his advices have been a very important support to my work and to my scientific growth in general. I am grateful to Dr. Dhiraj Patil from IIT Mandi for fruitful discussions during the last phase of this work.

Furthermore, I appreciate the support and help given by staff members of IIT Mandi. I would like to extend my thanks to the colleagues in our research group, Subit, Atendra, Ramakant and Sudeb. Special thanks to Chanderkant, Anshul, Ashish Joshi, Kothari, Manoj, Amit from Ghanauli and Jiten from Karsog for their friendship and great memories.

Financial support from University Grants Commission is also acknowledged.

Finally, I take this opportunity to convey my gratefulness to my mom and dad whom I am really proud of. Without them I would never have enjoyed so many opportunities. Thank you both for giving me strength to reach for the stars and chase my dreams.

(Hari Vansh Rai Mittal)

ABSTRACT

Keywords: *HOC Scheme, Cylindrical Polar Coordinates, Non-Uniform Polar Grid, Navier-Stokes Equations, Rotationally Oscillating Circular Cylinder, von-Kármán Vortex Street, Immersed Interface, Level Set, Elliptic Equations, Discontinuous Coefficients, Cartesian Grid, Lagrange Polynomial Interpolation, Stokes Flows, Stefan's Problem.*

The purpose of the work presented herein is to introduce novel numerical approaches, which can be used for various complex multi-phase fluid interface problems. We first aim to study the vortex shedding phenomena for the case of flow past a rotationally oscillating circular cylinder, at the initial stages and for the fully developed flows for different Reynolds numbers. Navier-Stokes (N-S) equations are solved using recently developed Higher Order Compact (HOC) scheme on a body fitted, non-uniform polar grid in such a way that surface of the cylinder forms the inner boundary of the domain. This study provides insight into the wake characteristics associated with vortex shedding phenomena from a rotationally oscillating circular cylinder due to fluid-structure interactions and consolidates previous related observations, often unaccounted for, found in the published literature. Then, a new modified HOC based finite difference scheme is developed in polar coordinates to solve elliptic partial differential equations (PDEs) in domains with discontinuities due to the presence of circular interfaces. After validating this scheme by solving elliptic PDEs, it is extended to N-S equations for simulating the flow past an impulsively started circular cylinder. Body fitted, non-uniform polar grid in constructed in such a way that surface of the cylinder is considered as an interface immersed in the fluid. All the flow characteristics including the von-Kármán vortex street are accurately captured. The most significant contribution of this work, however, is the development of a new generalized finite difference scheme for solving problems with arbitrary shaped fixed interfaces on a non body fitted, uniform Cartesian grid. Extensive validation is carried out by solving elliptic equations in two and three dimensions followed by its extension to solve incompressible, 2D Stokes flows. It produces excellent results for pressure term even if in a coarser grid like 32×32, and the results are significantly better than those of existing methods. Linear regression analysis is used to find order of accuracy of the scheme using grid refinement which shows that the proposed scheme has average second order accuracy. This idea is then extended to solve moving interface problems with discontinuities across time dependent moving interfaces. An unsteady, non linear general partial differential equation and Stefan's problem are solved with a moving interface in 1D. In 2D, heat equation is considered on a square domain with a circular interface whose radius is continuously changing with time. Crank-Nicolson (CN) type discretization is compared with a recently developed HOC scheme. HOC scheme is found to produce better results and the order of accuracy is also more than that of CN scheme. The scheme shows nearly second order accuracy and good agreement with the analytical solution.

Contents