Modeling The Dynamics of Waterborne Diseases Under The Influence of Environmental Pollution

A Thesis Submitted

in Accordance with the Requirements

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DOCTOR OF PHILOSOPHY

By

SANDEEP SHARMA

(D12068)

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THESIS CERTIFICATE

This is to certify that the work contained in the thesis entitled **" Modeling The Dynamics of Waterborne Diseases Under The Influence of Environmental Pollution "** being submitted by **Mr. Sandeep Sharma (Enroll. No: D12068)** has been carried out under my supervision. In my opinion, the thesis has reached the standard fulfilling the requirement of regulation of the Ph.D. degree. The results embodied in this thesis have not been submitted elsewhere for the award of any degree or diploma.

Dr. Nitu Kumari (Supervisor)

Assistant Professor, School of Basic Sciences

IIT Mandi, Mandi, India

Email: nitu@iitmandi.ac.in

Declaration by the Research Scholar

I hereby declare that the entire work carried out in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of Dr. Nitu Kumari, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgments have been made wherever the work described is based on findings of other investigators.

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Signature:

(SANDEEP SHARMA)

ABSTRACT

For centuries, infectious diseases have challenged the survival and development of the society. Occurrence of a number of infectious diseases cause huge loss in terms of lives, money and resources. Waterborne diseases, that can be defined as a subclass of infectious disease, are transmitted due to the drinking of water contaminated with pathogens. The diseases (cholera, typhoid, giardiasis, shigella hepatitis A and hepatitis E etc.) that belong to this class are also a major contributor in all the above mentioned events. Apart from infectious diseases, environmental pollution also emerges as a major menace of the present time. Unfortunately, the factors involved in our day to day activities (e.g transportation, industry and household activities) are the major sources of various components of environmental pollution (like water, air and soil). To date, both of the above mentioned perils, i.e. infectious diseases and environmental pollution, are studied separately. However, there are evidences suggesting that regular exposure to pollutants affects the health adversely. Furthermore, it also paralyzes the immune system and increases the susceptibility towards different diseases.

In the present thesis we develop some deterministic mathematical models to study the impact of pollution on the spread of infectious waterborne diseases. The dynamics of waterborne diseases is complicated due to the possibility of multiple transmission pathways, i.e. water to person and person to person disease transmission. In the modeling process, we consider single and multiple transmission pathways. The expression of basic reproduction number, which is a threshold parameter and plays a key role in the disease dynamics, has been obtained for each proposed model. The disease free and endemic equilibrium points are also obtained for the proposed models. We also obtain the stability conditions, both local and global, for the disease free and endemic equilibrium points with the aid of dynamical systems tools and theory of differential equations. To obtain the global stability,

we use the Lyapunov method and geometric technique. Application of geometric technique is tedious since the models investigated are four dimensional. We also perform extensive numerical simulations to demonstrate the role of environmental pollution in the spread of infectious diseases. The significance of the present work lies in the fact that it unfolds an important new dimension in the disease modeling.

Keywords: Infectious diseases, Waterborne diseases, Environmental pollution, Mathematical models, Basic reproduction numbers, Disease free equilibrium, Endemic equilibrium, Backward bifurcation, Stability, Lyapunov function, Compound matrix, Geometric technique

Dedicated To

My family and

"Atharv"

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(Sandeep Sharma)

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