MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS AND THEIR APPLICATION TO FINANCIAL PORTFOLIO OPTIMIZATION

A THESIS

Submitted for the Degree of

DOCTOR OF PHILOSOPHY

in the School of Basic Sciences

by

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Dedicated to my Parents and Teachers.

THESIS CERTIFICATE

I hereby certify that the entire work in this Thesis has been carried out by Mr. Suraj Shankarlal Meghwani (Enrollment No: D12071) under my supervision in the School of Basic Sciences, Indian Institute of Technology Mandi, and that no part of it has been submitted elsewhere for any Degree or Diploma.

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DECLARATION BY THE RESEARCH SCHOLAR

I hereby declare that the entire work embodied in this Thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of Dr. Manoj Thakur, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgements have been made wherever the work described is based on finding of other investigators.

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ACKNOWLEDGMENTS

First of all, I would like to thank my supervisor, Dr. Manoj Thakur, for his guidance and inspiration during my research. I appreciate his patience at times when I took considerable time for the analysis. He also allowed and encouraged me to work on the allied problems without any objection thereby providing me all necessary freedom. Without his insightful discussions, constant support and motivation, this work would not have been completed. I also extend my appreciation to my doctoral committee whose suggestions improved the quality of this work. I would also like to express my heartfelt gratitude to my master's supervisors and teachers, Prof. Mihir Arjunwadkar, Dr. Niruj Mohan Ramanujam, Dr. Sukratu Barve and Prof. Jayaram Chengalur whose inspiration initiated my academic career and helped me to develop strong work habits and mathematical skills.

I take this opportunity to sincerely acknowledge the Ministry of Human Resource Department (MHRD), Government of India, New Delhi, for providing financial assistance through Indian Institute of Technology Mandi which buttressed me to perform my work comfortably. I am also grateful to the organizers of several enlightening workshops and summer schools in India. With special mention to Indo-French Centre for Applied Mathematics (IFCAM) summer school in 2015 at IISc Bangalore, Workshop in Financial Mathematics in 2012 at IIT Guwahati and Advanced Workshop on Variational Analysis and Optimization in 2015 at IIT Gandhinagar.

I would like to thank my colleagues Sandeep Sharma (& family) and Pankaj Narula for their excellent support during the good and challenging time in IIT Mandi. I would also like to take this opportunity to acknowledge the support of my collaborators – Dr. Syed Abbas, Dr. Jai Prakash Tripathi, Dr. Hemant Jalota, and Dr. Deepak Kumar. I also wish to thank institute's administrative and security staff for unfailing support and assistance during my stay at IIT Mandi.

Finally and most importantly, I would like to express my deepest gratitude to my parents for all their unconditional love and moral support.

Thank you all for the encouragement!

Suraj Shankarlal Meghwani

ABSTRACT

Markowitz, in his seminal work, posited the concepts of portfolio optimization and diversification that had been instrumental in the understanding of financial markets. Markowitz formulated the portfolio selection problem as an optimization problem in which the risk, measured through the variance of the portfolio returns, is minimized at a given level of desirable expected portfolio return. Undoubtedly, Markowitz's theory had an everlasting impact on both academic research and financial industry. The mean-variance optimization problem serves as a starting point in the actual practice. Even though the foundational framework of the Markowitz theory remains same but in practice, many institutional rules, investor specific guidelines or portfolio manager's personal preferences must be included in the optimization framework. To include these practical considerations, classical mean-variance portfolio optimization problem is extended in several directions. Generally, when institutional rules and/or investor's preferences are reflected in the portfolio optimization problem, it leads to complicated constraints that pose a serious challenge to the solution algorithms.

Markowitz theory has a myopic viewpoint, i.e., it was developed for portfolio construction in a single period. Given the inputs, Markowitz optimization model provides optimal allocations of assets which are assumed to hold at the end of the investment horizon. However, in real-scenarios, risk-return characteristics are continuously changing in the market. Hence asset allocations are need to be re-optimize and rebalance with respect to changing market conditions. Further, rebalancing of a portfolio incurs transaction costs which are inevitable. These costs, when considered in an ex-post manner, often lead to inefficient portfolios. Hence, consideration of transaction costs at the time of the portfolio optimization and rebalancing becomes essential.

Transaction costs are generally modelled as non-linear, discontinuous function of the change in the volume of assets in the portfolio. Hence, classical optimization packages are unable to handle portfolio optimization models involving these costs. Recently, **M**ulti **O**bjective **E**volutionary **A**lgorithm (MOEA)s has been emerged as a promising alternative for handling optimization problems having complex objectives and constraints. In the present study, MOEAs are investigated, and a modified decomposition based MOEA is proposed to achieve better convergence-diversity statistics than several state-of-the-art and recent MOEA over benchmark **M**ulti-objective **O**ptimization **P**roblems (MOPs).

In the second part of the present work, MOEAs are adapted for solving portfolio optimization models involving practical constraints. We have designed repair algorithms for dealing with practical constraints and established their effectiveness in the context of MOEAs. The adaptations of MOEAs along with the proposed candidate generation mechanism and repair algorithm handle all the constraints without requirement of any traditional constraint handling procedure.

A tri-objective portfolio optimization model with total transaction costs as one of the objectives along with risk and return objectives is proposed. Inclusion of cost objective introduces several cost-related equality constraints, that are difficult to handle by traditional constraint handling approaches used in MOEAs. Hence, a specialized repair algorithm is designed for handling these equality constraints. The proposed repair algorithm is also amenable to a larger class of cost models used in realistic scenarios. The study investigates portfolio rebalancing problem involving several practical constraints along with different risk-measures, viz., variance, **V**alue-**a**t-**R**isk (VaR), and **C**onditional **V**alue-**a**t-**R**isk (CVaR)

In a nutshell, this study establishes that the designing specialized problem-specific algorithms for dealing with realistic constraints is an effective alternative. Further, proposed adaptations of MOEAs are also advantageous in the context of portfolio optimization and rebalancing. Although repair algorithms designed in this study can fit in any general procedure of population-based algorithms, we found out adapted **N**on-dominating **S**orting **G**enetic **A**lgorithm (NSGA)-II achieve superior empirical results in comparison to other state-of-the-art and some recent MOEAs used in the comparisons.

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