

**UNSTEADY FLOW SEPARATION FROM THE  
SURFACE OF SOLID CYLINDERS IN LAMINAR  
SHEAR FLOW: A STRUCTURAL BIFURCATION  
ANALYSIS**

*A THESIS*

*submitted by*

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*for the award of the degree*

*of*

**DOCTOR OF PHILOSOPHY**



**SCHOOL OF BASIC SCIENCES**

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**DECEMBER, 2018**

*Dedicated*

*To*

*My Parents*



## THESIS CERTIFICATE

This is to certify that the thesis titled “**UNSTEADY FLOW SEPARATION FROM THE SURFACE OF SOLID CYLINDERS IN LAMINAR SHEAR FLOW: A STRUCTURAL BIFURCATION ANALYSIS**”, submitted by **ATENDRA KUMAR**, to the Indian Institute of Technology Mandi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma.

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## **Declaration by the Research Scholar**

I hereby declare that the entire work embodied in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of **Dr. Rajendra K. Ray**, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgments have been made wherever the work described is based on finding of other investigators.

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**(ATENDRA KUMAR)**



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**(Atendra Kumar)**





# ABSTRACT

**Keywords:** *Higher Order Compact(HOC) Scheme, Cartesian Coordinates, Polar Coordinates, Navier-Stokes Equations, Square cylinder, Circular cylinder, Structural Bifurcation, Vortex Shedding, Unsteady flow separation, Shear Flow.*

The study of shear flow past square and circular cylinders is investigated by two-dimensional (2-D) numerical simulations. The unsteady flow separation leading to vortex shedding from cylinders surface is studied using topological aspects based structural bifurcation analysis. A fourth order accurate in space and second order accurate in time higher order compact (HOC) finite difference scheme is used to solve unsteady, incompressible Navier-Stokes (N-S) equations in uniform Cartesian coordinates. The vortex shedding phenomenon and wake development are studied by vorticity contours and streakline patterns, lift and drag coefficients, phase diagrams, etc. This study provides the insight into structural bifurcation caused by unsteady flow separation from cylinder surface and consolidates previous related observations, often unaccounted for, found in the published literature. All the flow characteristics including the von-Kármán vortex street are accurately captured.

For flow past single square cylinder, the Reynolds number( $Re$ ) ranging from 100 to 500 and shear parameter values ( $K$ ), ranges from 0.0 to 0.4 are considered. It is observed that vortex shedding and wake development behind the square cylinder significantly depend on  $Re$  and  $K$ . It is also observed that vortex shedding completely suppressed for a critical  $K$  value for particular Reynolds number. The exact location and time of occurrence of structural bifurcation points from the surface of square cylinder are extensively studied using topological aspects based structural bifurcation analysis. Further, the flow characteristics are verified by describing the existence of saddle points. The computations for shear flow past two and three square cylinders in various arrangements are carried out for  $Re = 100, 200$  and  $K = 0.0$  to  $0.4$  to study the interference of vortex shedding in between and behind the square cylinders. For the case of two square cylinders, the gap ratio ( $s/d$  where  $s$  is the gap between cylinders,  $d$  is the length of cylinder side) ranges from 0.5 to 3.0. The vertical gap( $s$ ) between downstream cylinders for shear flow past three square cylinders in vee shape arrangement is considered from 0.6 to 3.0 with fixed horizontal gap  $2d$ , between the upstream and downstream cylinders. We have shown that the values of  $s/d$  and  $K$  signifi-

cantly affect the location and time of appearance of bifurcation points. A details study on fully developed flows are carried out in terms of streaklines, vorticity contours, lift and drag coefficients, phase diagrams, centerline velocity fluctuations, etc., for two identical square cylinders in series arrangement and side-by-side arrangement, and three identical cylinders in vee shape arrangement. A comprehensive study of the exact location and time of occurrence of structural bifurcation points is carried out using structural bifurcation analysis for above mentioned two cylinder and three cylinder cases. The existence of saddle points is also studied for these flow problems. Many new flow phenomena are observed for these cases for the first time. Finally, the computations and structural bifurcation analysis are extended to shear flow past circular cylinder at  $Re = 100, 200$  and  $K = 0.0$  to  $0.2$ . In this case the governing equations are considered in cylindrical polar coordinates and HOC scheme is used accordingly for discretization. The flow phenomenon for fully developed flow is visualized in terms of vorticity contours, streakline patterns, lift and drag coefficients, phase diagrams, etc. We conclude that frequency of the vortex shedding is inversely proportional to the shear rate. Finally, we study the exact location and time of occurrence of structural bifurcation points, for initial stages of flow, as well as fully developed flow, for all  $K$  values mentioned above, using structural bifurcation analysis. Through this study many new things are observed, for the first time, for this type of flow. Overall, this study offers some notable contribution to this field and opens the new direction of future study.

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