Characteristics of double-diffusive finger evolution: Numerical, Analytical and Experimental Study

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by

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Dedicated to

My parents for their immense support and love.

Declaration

I hereby declare that the entire work embodied in this thesis is the result of investigations carried out by me in the **School of Engineering, Indian Institute of Technology Mandi**, under the supervision of **Dr. Atul Dhar and Dr. Om Prakash Singh**, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgements have been made wherever the work described is based on finding of other investigators.

Place: Mandi **Faria Rehman**

Date:

Thesis Certificate

This is to certify that the thesis titled **Characteristics of double-diffusive finger evolution: Numerical, Analytical and Experimental Study**, submitted by **Faria Rehman**, to the Indian Institute of Technology Mandi for the award of the Degree of Doctor of Philosophy is a bonafide record of the research work carried out by her under my supervision. The content of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any Degree or Diploma.

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ABSTRACT

The main focus of the research work reported in this thesis is to study the characteristics of double-diffusive finger evolution and effective parameters that govern them. The double-diffusive instabilities, salt fingers, arises in oceans when hot and salty water lies over the cold and fresh water of greater density. Convection is driven by the varying diffusivities of the two components that contribute in the density variation, that is heat (T) and salinity (S), where diffusivity of heat is hundred times faster than the salt. Faster diffusing component (T) stabilize the system whereas slower diffusing component (S) destabilize the system with overall density stratification remaining stable. The convection in this state takes the form of rising and sinking fingers known as salt fingers. Salt fingers are found to be highly interactive and have an extensive contribution to vertical mixing, nutrients up-welling and transport of heat and salt fluxes in oceans. In some tropical oceans, it is considered as the major source of transportation of heat and salt fluxes which is higher than turbulent mixing. For example, in tropical western Pacific Ocean, the estimated flux and associated vertical diffusivity due to double diffusion is approximately one order of magnitude higher for temperature and two orders of magnitude higher for salinity compared to values calculated from a turbulence model. Also, in several provinces of Atlantic, Pacific and Indian oceans, nitrate diffusion mediated by salt fingers is responsible for 20% of the new nitrogen supply. The scope of double-diffusion convection is not restricted to oceanography; it is widely spread in other fields also, like stellar and planetary dynamics and evolution, crystal growth, magma chambers, geology etc.

To explore the salt finger phenomenon, a non-dimensional density ratio (R_ρ) is determined which is a measure of the degree of compensation between temperature and salinity gradients in terms of their effects on density stratification. For fingers to form in the heat-salt system, $1 < R_\rho < \tau^{-1}$. Here, τ is the diffusivity ratio of the slowest to the fastest diffusing components. It is suggested by literature that finger formation takes place at low R_ρ in oceans and most distinctive fingers are formed when R_ρ approaches unity. Many linear theories have been proposed to understand the finger behaviour as a function of density stability ratio (R_ρ). Theories of Stern

(1960), Schmitt (1979, 2011) and Kunze (1987) are prominent and acceptable in the scientific community. However, these theories are based on various assumptions which have some repercussions. One of the outcomes of these assumptions is lack of accuracy when $R_\rho \rightarrow 1$. Unbounded results are obtained at neutral buoyancy ($R_\rho = 1$). Whereas many experimental studies in literature clearly indicate the formation of salt fingers at $R_\rho = 1$. Now the question arises that why this kind of unusual behaviour is observed only at low R_ρ, what is the physical interpretation of the assumptions that lead to the unboundedness at R_ρ and how finger behaviour is different from other R_ρ, let say $R_\rho = 2$. We addressed this issue in this thesis and have shown that these assumptions in the previous theories are invalid in a certain range of governing parameters. Moreover, it is believed in the scientific community that wide fingers transport large fluxes of heat and salinity compared to a thin fingers. Theories proposed on this basis are also scrutinized. New insights into the physical understanding of the assumptions behind the previous linear theories are discussed with the help of numerical simulation. A numerical study is carried out to study the assumption taken by these linear theories that wide fingers transport large fluxes of heat and salt compared to thin finger. Simulations were run for the wide range of Rayleigh number, from 7×10^3 to 7×10^8 and it was observed that the flux ratio of the narrower fingers is more than that of the wide fingers, which is contrary to the assumptions taken by these linear theories.

In the context of recent observations, which states that fingers growth rate, kinetic energy, evolution pattern, finger width etc. were demonstrated to be a strong function of Rayleigh numbers and weak dependence on R_ρ, it would be difficult to predict finger characteristics from the previous linear theories, which are only the strong function of R_ρ and the role of Rayleigh number is not considered. Therefore, a new theory has been developed for growth rate from the linearized governing equations with explicit dependence on Rayleigh numbers, density stability ratio, Schmitt number, Prandtl number and diffusivity ratio (of the components contributing to a density of the system). Expressions for power-law equations of maximum growth rate are also derived as a function of various non-dimensional parameters. The model predicts the finger characteristics reasonably well such as wavelength and fluxes reported by previous investigators. In the limit of parameter value approaching unity i.e. ($Ra_T, R_\rho \rightarrow 1$), the growth rate model collapse into a single curve and follow Schmitts theory (1979). The predicted results of the new theory corroborate well with the data reported from the field measurements, experiments and numerical simulations.

Lately, a new finger regime has been observed in some experimental study, where finger convection occurs even for $R_\rho < 1$. Moreover, linear theories also indicated the unusual behaviour of salt fingers at neutral buoyancy ($R_\rho = 1$). But critical information such as convective structures and fluxes are still unknown as to what exactly happens at neutral buoyancy. There has been a comprehensive study of salt finger convection at $R_\rho > 1$ but scarce literature exists that explored finger behaviour at R_ρ in a large range of governing parameters. The intriguing behaviour of fingers at neutral buoyancy motivated us to further investigate it. In this thesis, we present the unexplored finger convection numerically at various ranges of parameters where the initial system is at $R_\rho = 1$. We have solved numerically the partial differential equations governing the continuity of mass, momentum, energy and species in two-dimension. We have considered a two-layer system similar to the laboratory setup. A series of simulations have been conducted in the heat-salt system at a fixed R_ρ equal to one for Rayleigh numbers ranging from 7×10^3 to 7×10^8. It is noticed that salt fingers characteristics and evolution pattern vary drastically with the change in Rayleigh number. We have reported finger structures, mean profiles and kinetic energy at a wide range of Rayleigh number. To provide useful insight of the system, i.e. how fingers transport heat and salt fluxes vertically, the layer properties are investigated in details by analysing upper and lower layer of the system as the fingers run down. The ratio of heat to salt fluxes called flux ratio (R_f) has been studied at $R_\rho = 1$ and the most common belief of scientific community that when $R_\rho \rightarrow 1 \implies R_f \rightarrow 1$ has been investigated and it is observed that this condition is true only for high Rayleigh number. A new insight is developed on the finger behaviour at neutral buoyancy which is further investigated experimentally.

Recent experimental study of Hage and Tilgner (2010) has revealed a surprising new finger regime where thin fingers form even for a small stabilizing temperature gradient, results in $R_\rho < 1$, which is not considered as a usual domain for finger formation. Finger formation below neutral buoyancy has not been studied much in the past much. In this thesis, we have explored this new regime of finger convection numerically for the first time at a large range of governing parameters where the initial system is at $R_\rho = 1/10$. Finger evolution, flux ratio, kinetic energy, layer properties, density variation and R_ρ variation with time is examined for this regime and some first-hand results are reported. At low Rayleigh number, initial layer properties of average concentration and temperature exceed the mean value. However, at high Rayleigh numbers, layer properties do not change as rapidly and remains near the initial layer properties. Moreover, thin fingers were seen only at high Rayleigh number however at low Rayleigh number, scarcely any fingers formed.

Experimental study of double-diffusive salt finger convection is conducted in a Hele-Shaw cell to investigate the effect of two most significant governing parameters, Rayleigh number and density stability ratio. Finger characteristics like structure, evolution time, growth rate and finger width at variable Rayleigh number and density stability ratio are investigated. The two layer system was formed using salt and sucrose solution ($\tau = 1/3$). Rhodamine-B was used as a dye for fluid visualization which was added to a salt solution. Rayleigh number and density stability ratio were systematically changed one at a time by controlling the concentration of the solute. Experiments are conducted for a wide range of Rayleigh number, $O(10^3) - O(10^6)$, for $R_\rho = 1 - 1.5$. Salt finger characteristics like finger structures, growth rate, finger width and evolution time are reported and it is observed that they strongly vary as a function of Rayleigh number. It is observed that salt finger behaviour observed at neutral buoyancy ratio in experimental study matches with that of the numerical simulations. Bulbous tips are observed at neutral buoyancy ratio at high Rayleigh number in both experiments and numerical simulations. However, such behaviour is delayed at low Rayleigh number.

Contents