

Study of Laminar Micro-Convective Flow with Variable Fluid Properties

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Doctor of Philosophy

'by'

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“Dedicated to my beloved parents & family”

For their love, endless support, encouragement & sacrifices

ABSTRACT

Over the last two decades, the rapid advances in micro-fabrication techniques have provided the capability to batch fabricate micro-devices quickly and inexpensively. Therefore, the use of high-density, high power, and high-speed micro-devices are increasing day by day that leads to widespread interest in the micro-fluid mechanics and the need for both comprehensive and systematic investigation of the fundamental aspects of these phenomena. Flows in such devices are subjected to high-temperature gradients and operate at low Reynolds numbers. These flows exhibit substantial variations in fluid properties, viz. Density (ρ), Dynamic viscosity (μ), Thermal conductivity (k), and Specific heat at constant pressure (C_p). The present study investigates the influence of variations in fluid properties (a Non-rarefaction effect) on liquid and gas micro-convection. Some deviations at the micro-scale flow from conventional theory have been reported in the past, but the causes of these deviations are not comprehensively and systematically explained or justified. Thus, the objective of this thesis is to understand flow deviations due to variations in fluid properties, which can help in the design of micro-channel heat sinks of micro-devices or cooling system of gas turbine blades. The present investigations can be summarized as follow:

The physical effects of fluid property variations on flow and thermal development in micro-channel are numerically analyzed. The effects of temperature-dependent density $\rho(T)$, viscosity $\mu(T)$, and thermal conductivity $k(T)$ variations on single-phase laminar forced convection are investigated. The problem is especially simulated for hydrodynamically and thermally developing water flow in micro-channel with no-slip, no-temperature jump, and constant wall heat flux boundary conditions. The present research, by means of numerical results, analyzes the influence of $\rho(T)$, $\mu(T)$, and $k(T)$ variations on Fanning friction factor (f_f), Poiseuille number (Po), dimensionless wall velocity gradient $(\partial\bar{u}/\partial\bar{r})_w$, static gauge pressure

drop (Δp_s) and Nusselt number (Nu). The effects of temperature-dependent viscosity variation on fully developed flow through a micro-channel are also numerically investigated in this thesis. The effects of $\mu(T)$ variation are able to couple the velocity and temperature fields. Therefore, the velocity and temperature profiles vary qualitatively along the micro-flow. Due to $\mu(T)$ variation, the concept of *flow undevelopment* (the reverse process of flow development) is observed in the flow regime.

The Chilton-Colburn analogy is very helpful for evaluating the heat transfer in internal forced flows. In the present thesis, the validity of Chilton-Colburn analogy between $St \cdot Pr^{2/3}$ and f_f is re-examined for laminar micro-convective flow with variable thermophysical fluid properties. It is observed that the Chilton-Colburn analogy is valid only for that portion of the flow regime, where $St \cdot Pr^{2/3}$ decreases with decreasing f_f . The validity of Chilton-Colburn analogy is also verified by the inverse dependence of Reynolds number (Re) with f_f . Two modified non-dimensional parameters " $\Pi_{S\mu T}$ and Π_{SkT} " are emerged from the non-dimensional form of 2D, steady state, incompressible, pure continuum-based, laminar conservation of momentum and energy equations respectively. Additionally, the role of $\Pi_{S\mu T}$ and Π_{SkT} in flow friction is also investigated.

The present study also investigates the frictional and heat transfer characteristics of water flowing through a circular micro-channel with variable fluid properties. The computational analysis reveals the importance of physical mechanisms due to variations in thermophysical fluid properties such as viscosity $\mu(T)$, thermal conductivity $k(T)$ and density $\rho(T)$ and also their contribution in the frictional and heat transfer characteristics. Various combinations of thermophysical fluid properties have been used to find their effects on fluid friction. It is observed that the fluid friction attains the maximum value in the vicinity of the inlet and diminishes along the flow. Reynolds' analogy helps to find the flow regime in which heat transfer increases while shear stress decreases for thermophysical fluid properties.

Therefore, the present study also numerically investigates the validity of Reynolds' analogy between Stanton number (St) and Fanning friction factor (f_f) for micro-convective water flow considering combined variations in fluid properties such as temperature-dependent density, viscosity, and thermal conductivity. It is proposed that the Reynolds' analogy is valid when St increases with a decrease in f_f for thermophysical fluid properties. Three modified non-dimensional parameters ($\Pi_{St\rho T}$, $\Pi_{St\mu T}$, and $\Pi_{Stk T}$) appear from the non-dimensionalization of the governing conservation equations. The dependence of the friction factor on these parameters is examined with the help of dimensional analysis. It is also noted that the effects of variation in fluid properties on the convective heat transfer coefficient and Nusselt number are significant for micro-convective flow.

The physical effects of variable fluid properties on heat transfer and frictional flow characteristics of laminar gas-micro-convective flow are also investigated in this research. The physical effects induced due to variations in air density with pressure and temperature, and gas viscosity, thermal conductivity, and specific heat with temperature are analyzed. Numerical results reveal that the heat transfer and frictional characteristics of laminar gas micro-flow are drastically affected by these physical effects. Hence, the present research suggests that these physical effects need to be well considered in the applications of laminar gas-micro-convection based on large temperature gradients, e.g. the design of micro-channel heat sink, and the flow cannot be generally considered as constant property flow, as in conventional channels.

Keywords: Micro-channels, Fully developed flow, Heat transfer characteristics, Frictional flow characteristics, Variable property effects, Dimensional analysis

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TABLE OF CONTENTS

ABSTRACT	i
ACKNOWLEDGMENTS	iv
LIST OF PUBLICATIONS	v
TABLE OF CONTENTS	vi
LIST OF FIGURES	x
LIST OF TABLES	xiii
NOMENCLATURE.....	xiv
CHAPTER 1	1
Introduction.....	1
1.1 General Introduction	1
1.2 Liquid Flows in Micro-Channel.....	6
1.2.1 Important deviations in micro-flow characteristics.....	7
1.2.2 Important deviations in micro-convection characteristics.....	8
1.3 Gaseous Flows in Micro-Channel.....	8
1.4 Novelty and Uniqueness of the Present Study	11
1.5 Motivation for the Study	12
1.6 Objective and Scope of Investigation	13
1.7 Outline of the Thesis	15
References.....	16
CHAPTER 2.....	19
Literature Review	19
2.1 Introduction.....	19
2.2 Review of Micro-Convection.....	19
2.3 Scaling Effects in Micro-Flows	24
2.3.1 Variable fluid properties effects	24
2.3.2 Axial conduction effects.....	27
2.3.3 Viscous dissipation effects	29
2.3.4 Compressibility effects.....	31
2.3.5 Surface roughness effects	32
2.4 Special Role of Numerical Methods in Property Variation Studies	33
2.5 Conclusions from Literature Survey.....	34
References.....	35
CHAPTER 3.....	40

Physical Effects of Variable Thermophysical Fluid Properties on Flow and Thermal Development	40
3.1 Introduction.....	40
3.2 Physical Model and Boundary Conditions	41
3.2.1 Boundary conditions	42
3.2.2 Mathematical formulation	43
3.2.3 Formulas used in the simulation.....	44
3.3 Computational Domain and Numerical Solution.....	44
3.3.1 Grid testing and validation	45
3.4 Results and Discussion.....	47
3.4.1 Entrance effect on velocity and temperature fields with constant fluid properties	48
3.4.2 Effects of property variation coupled with entrance effect on velocity field	52
(i) Role of $\rho(T)$ variation	52
(ii) Role of $\mu(T)$ variation.....	54
(iii) Role of $k(T)$ variation.....	58
3.4.3 Effects of property variation coupled with entrance effect on temperature field	63
(i) Role of $\rho(T)$ variation	63
(ii) Role of $\mu(T)$ variation.....	66
(iii) Role of $k(T)$ variation.....	68
3.5 Conclusions.....	69
References.....	70
CHAPTER 4.....	72
Effects of Temperature-Dependent Viscosity Variation on Fully Developed Laminar Micro-Convective Flow	72
4.1 Introduction.....	72
4.1.1 Scope of investigation	73
4.2 Problem Formulation	74
4.2.1 Re-examination of the Chilton-Colburn analogy for variable viscosity fluid.....	75
4.2.2 Mathematical modelling of governing conservation equations for variable viscosity fluid	77
4.2.3 Physical model of problem	80
4.2.4 Boundary conditions.....	81
4.3 Computational Domain and Numerical Methodology.....	82
4.3.1 Grid independence test.....	83
4.4 Results and Discussion.....	84
4.4.1 Thermophysical properties for heat transfer	84

4.4.2 Effect of $\mu(T)$ variation on fluid flow.....	85
4.4.3 Effect of $\mu(T)$ variation on heat transfer	94
4.5 Importance of modified non-dimensional parameter, $II_{S\mu T}$	96
4.6 Conclusions.....	99
References.....	100
CHAPTER 5.....	102
Numerical Reexamination of Chilton-Colburn Analogy for Variable Thermophysical Fluid Properties.....	102
5.1 Introduction.....	102
5.1.1 Scope of investigation	103
5.2 The Chilton-Colburn Analogy Revisited for Thermophysical Fluid Properties.....	103
5.2.1 Physical model of problem	105
5.2.2 Mathematical modelling of governing conservation equations for thermophysical fluid properties.....	105
5.2.3 Boundary conditions.....	108
5.3 Computational Domain and Numerical Methodology.....	109
5.3.1 Grid independence test.....	109
5.4 Results and Discussion.....	109
5.4.1 Thermophysical properties for heat transfer	109
5.4.2 Significance of the modified non-dimensional parameters, $II_{S\mu T}$ and II_{SKT}	119
5.5 Conclusions.....	122
References.....	124
CHAPTER 6.....	125
Frictional and Heat Transfer Characteristics of Micro-Convective Flow for Variable Fluid Properties.....	125
6.1 Introduction.....	125
6.1.1 Objective and scope of investigation	127
6.2 Problem Formulation and Description	128
6.2.1 Governing equations incorporating variations in thermophysical properties	129
6.2.2 Formulas used in the computations.....	132
6.2.3 Boundary conditions.....	133
6.3 Computational Domain and Numerical Solution.....	134
6.3.1. Grid independence test	134
6.4 Results and Discussion.....	134
6.4.1 Thermophysical properties of water	135
6.4.2 Frictional characteristics.....	136

6.4.2.1 Effect of variable fluid properties on f_f and Po	137
(1) Effect of $\mu(T)$ variation	137
(2) Effect of $\mu(T)$ and $k(T)$ variations	138
(3) Effect of $\mu(T)$, $k(T)$ and $\rho(T)$ variations	138
6.4.2.2 Effect of thermophysical fluid properties on the relationship of Re and f_f , Po and Re	142
6.4.2.3 Effect of thermophysical fluid properties on pressure drop	146
6.4.3 Re-examination of Reynolds' analogy for variable fluid properties	148
6.4.4 Significance of the modified non-dimensional parameters, Π_{SpT} , Π_{SmT} , and Π_{SKT}	156
6.4.5 Heat transfer characteristics.....	162
(1) Effect of $\rho(T)$ variation.....	162
(2) Effect of $\mu(T)$ variation	164
(3) Effect of $k(T)$ variation.....	167
(4) Effect of combined $\rho(T)$, $\mu(T)$, and $k(T)$ variation	168
6.4.6 Effect of variation in thermophysical properties on h	169
6.5 Conclusions.....	174
References.....	176
CHAPTER 7.....	178
Physical Effects of Variable Fluid Properties on Laminar Gas-Micro-Convective Flow	178
7.1 Introduction.....	178
7.2 Governing Equations	181
7.2.1 Boundary conditions.....	182
7.2.2 Solution methodology	183
7.3 Results and Discussion.....	184
7.4 Conclusions.....	190
References.....	191
CHAPTER 8.....	194
Concluding Remarks and the Scope for Future Research.....	194
8.1 Summary of Present Investigation	194
8.2 Scope for Future Research.....	199
Appendix-A.....	201