# Study of dynamic equations on time scale with applications

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*By*

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*to the*

**SCHOOL OF BASIC SCIENCES**

**INDIAN INSTITUTE OF TECHNOLOGY MANDI**

*Dedicated To*

*My supervisor for his constant support and encouragement,*

*and to my parents, brother and*

*sister for their endless love, support and*

*encouragement*


# Declaration by the Research Advisors

It is certified that the work contained in the thesis entitled **"Study of dynamic equations on time scale with applications"** being submitted by **Mr. Shekhar Singh Negi (Enroll. No: D15027)** has been carried out under our supervision. In my opinion, the thesis has reached the standard fulfilling the requirement of regulation of the Ph.D. degree. The results embodied in this thesis have not been submitted elsewhere for the award of any degree or diploma.

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I **Shekhar Singh Negi**, hereby declare that the entire work embodied in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of **Dr. Syed Abbas** and **Dr. Muslim Malik**. Also, it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgments have been made wherever the work described is based on the finding of other investigators.

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# ABSTRACT

By the dynamic equations on time scale, we mean an equation that keeps the equations of continuous, discrete and quantum calculus within themselves in the same equation. Here, the time scale is a non-empty closed subset of real numbers. Moreover, the significance of this equation is very much evident in those circumstances where we need to deal with differential and difference equations together. By keeping the applications under observation, a lot of studies of several orders of this equation have been done by many authors.

Through this thesis, we present several oscillatory results for the first and second-order dynamic equations on time scale. Our studies are more general to the studies given in the literature. Furthermore, we establish the results by using less restrictive conditions as compared to the existing conditions in the literature. These conditions are easy to verify and implement. As an application, by means of coincidence degree theory, we establish the existence of positive periodic solution of the general N-prey and M-predator model on time scale. On the other hand, we also establish a few sufficient conditions for oscillation of the $p$-Laplacian dynamic equation on time scale for which we use a relaxed technique that compliments the existing techniques to prove oscillatory results. Further, we also use the Riccati transformation technique to transform second-order dynamic equations into the first-order dynamic equation. Furthermore, we derive some important inequalities and directly utilize the use of a well-known Young's inequality for some of oscillatory results. In addition, we make the conditions of our findings such that we can easily demonstrate the well- known Kamenev and Philos-type oscillation criteria for our dynamic equations on time scale. Besides, our contribution is not limited to this, we provide a new trend of finding a derivative of continuous, discrete and quantum calculus. This derivative is denoted by the name "black-delta" (symbol ▲) -derivative on time scale. Through this derivative, we put together the usual and discrete derivatives at the same time. Further, it

has several applications in various fields, for instance, engineering, population dynamics, biology, economics, social-sciences and quantum physics, etc. Some fundamental results, associated with this derivative, are presented. From the point of application purposes, a necessary and sufficient condition for this derivative is also provided. Moreover, a drastic connection between the well-known Hilger derivative and new derivative on time scale is demonstrated which makes our outcome better in the comparison to Hilger-derivative on some time scale. Furthermore, some crucial examples are presented so that we could shed light on the practicability and effectiveness of our results.

**Keywords**: Time scale, Oscillation, Dynamic Equation on Time-scale, Dynamic inequalities, Prey-predator model, Continuation Theorem, Coincidence Degree Theory, Periodic solution, Delay Dynamic Equation, Riccati technique.

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