

STUDY OF HYBRID OPTOMECHANICAL SYSTEMS

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CERTIFICATE

This is to certify that the thesis titled "**STUDY OF HYBRID OPTO-MECHANICAL SYSTEMS**", submitted by **Manoj Das** to the Indian Institute of Technology, Mandi, for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma.

Dr.P.K.Pathak

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DECLARATION BY THE CANDIDATE

I hereby declare that the entire work embodied in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of **Dr.P.K.Pathak** that it has not been submitted elsewhere for any degree or diploma.

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Abstract

Quantum optomechanics has roots in early attempts to develop gravitational radiation detectors using the elastic deformations of large high Q mechanical resonators and optical interferometers with moving end mirrors. Pioneering theoretical work was performed by Braginsky, Caves and others. After the development of advanced micromechanical and nanomechanical devices Quantum Optomechanics provide us to probe extremely tiny forces, often with spatial resolution at the atomic scale. The present work concerns with the different optomechanical systems with its application in cavity QED and lasing. This thesis contains six chapters.

In the introduction part we have given the basic properties and application of the optomechanical systems. Fabry-Perot cavity is the simplest optomechanical system, which contains an optical resonator and a mechanical resonator. We have provided the characteristic properties of the Optical and mechanical resonator. Optomechanical coupling between mechanical resonator and the optical field is discussed. Two main consequences of this coupling "the optical spring effect"(an optically induced mechanical frequency shift and shift in the spring constant of the resonator) and "cold damping"(the optical field acts effectively as a viscous fluid that can damp the mirror motion and cool its center-of mass motion) are discussed. Several utilizations of optomechanical interaction is also provided in the introduction part.

In the second chapter we proposed measurement-based conditional generation of the superposition of mesoscopic states of a nanomechanical resonator. We consider a two-level quantum mechanical system (qubit) coupled with a nanomechanical resonator through strain mediation. The qubit is driven by two resonant lasers one of which is weak and another is strong. When both the lasers are on, an qubit state dependent displacement of the resonator has been seen. Measurement of the state of the qubit produces superposition states of the nanomechanical resonator. In the third chapter we provided basics of quantum walk and did some literature survey of quantum walk. Quantum walk of a resonator coupled with a qubit has been realized by driving the qubit with two resonant lasers(one is weak and another is strong). The TLS will act as Quantum coin and determine the direction of the steps taken by the resonator. The step size will depend on the weak laser intensity.

In the fourth chapter we have taken a doubly clamped nanomechanical resonator as a Hybrid optomechanical system. The quantum Langevin equations for the Quadrature operators has been derived. The quadrature operators are written as the sum of its steady state value and its fluctuations around its classical steady state value. Then Quantum Langevin equations for the fluctuating Quadrature operators are derived. Variance of the resonator position fluctuation can be obtained by integrating the position spectrum with respect to frequency, which can be used to determine its non classical properties.

In the fifth chapter we investigate lasing in system of two separated quantum dots embedded inside a single mode cavity. We discuss two cases, first when quantum dots are pumped incoherently second when a laser is applied to pump quantum dot coherently. The quantum dots get mutually coupled through interaction with common electromagnetic field, thermal photon bath and acoustic phonon bath. Thus effectively two quantum dots form a three level or four level system.

In the last chapter we have discussed the concluding remarks.

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