

**Soft computing approaches for portfolio optimization: An
empirical study**

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Dedicated to
My Parents, Brother
and
Wife

Declaration by the Research Scholar

I hereby declare that the entire work embodied in this Thesis is the result of investigations carried out by me in the School of Basic Science, Indian Institute of Technology Mandi, under the supervision of Dr. Manoj Thakur, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgments have been made wherever the work described is based on finding of other investigators.

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I hereby certify that the entire work in this Thesis has been carried out by Hemant Jalota, under my supervision in the School of Basic Science, Indian Institute of Technology Mandi, and that no part of it has been submitted elsewhere for any Degree or Diploma

Signature:

Name of the Guide: **Dr. Manoj Thakur**

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ABSTRACT

In Markowitz's modern portfolio analysis, an investor tries to find out the best possible asset allocation among various possible options with an objective to minimize the risk and maximize the profit. To make more realistic portfolio model, many modifications were incorporated by researchers in the standard Markowitz model. Introducing cardinality constraint in portfolio problem restricts the portfolio allocation to be a subset of all available assets. Though it helps in reduction of transaction cost but it changes the structure of the problem from continuous to a mixed integer non-linear problem. Further, restricting the allocation in each of the component of the portfolio to certain range transforms the problem having semi-continuous variable.

In modern portfolio theory the asset returns are assumed to be random variables. Many portfolio models in crisp and probabilistic environment have been proposed and analyzed in the past. Portfolio selection problems which include fuzzy variables are referred to as fuzzy portfolio selection problems. These problems use fuzzy measures to represent the uncertainty and vagueness involved in various parameters used to define the portfolio model. In real life scenarios, information is usually specified vaguely to investors that limits investors to devise a probabilistic model. Fuzzy set theory emerges as an alternative to model uncertainty where the vague inputs are easily quantifiable as fuzzy numbers. Assigning membership function to fuzzy event is an essential step while modeling the fuzzy portfolio problem. The methods stated in the literature to generate membership function for fuzzy event depend upon the information provided by experts.

Problem having semi-continuous variable are challenging to solve (especially for large financial markets) in both probabilistic and fuzzy settings. Classical techniques turn out to be fragile for solving these class of problems. Heuristic algorithms propose an alternate for solving these kind of problems. These class of algorithms have gained popularity as they have shown fairly good performance in many industrial applications.

The present study proposes and investigates a genetic algorithm to solve standard portfolio problem and its variants for solving portfolio optimization problem with semi-continuous variable. The results obtained using proposed algorithms are compared with other algorithms for solving the same class of problems. Further to model portfolio problems in fuzzy environment, an algorithm to generate membership function from the given

data is proposed. This algorithm does not depend upon experts guidance. Some new portfolio selection models using credibility measure in fuzzy environment are proposed. These models are designed to capture the uncertain nature of market and linguistic expressions used by investors to describe their aspiration values. To demonstrate the effectiveness of the proposed solution approach and applicability of the proposed models in practical applications of portfolio selection, an empirical study is conducted on data obtained from National Stock Exchange (NSE) India. The performance of models proposed are compared on the basis of Crediblistic Sharpe ratio defined in fuzzy context.

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