ANALYSIS OF NEURAL NETWORK MODELS OF INTEGER AND NON-INTEGER ORDER

A Thesis Submitted

in Accordance with the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

By

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THESIS CERTIFICATE

This is to certify that the work contained in the thesis entitled **"Analysis of Neural Network Models of Integer and Non-Integer Order "** being submitted by **Ms. Swati Tyagi (Enroll. No: D12072)** has been carried out under my supervision. In my opinion, the thesis has reached the standard fulfilling the requirement of regulation of the Ph.D. degree. The results embodied in this thesis have not been submitted elsewhere for the award of any degree or diploma.

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Declaration by the Research Scholar

I hereby declare that the entire work carried out in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of Dr. Syed Abbas, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgments have been made wherever the work described is based on finding of other investigators.

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ABSTRACT

The present thesis deals with the investigation of more interesting dynamics in various generalized models of artificial delayed neural networks of both integer order and fractional-order. The study provides insight into the detailed stability analysis of the various delayed models, such as existence and uniqueness of solutions to the network models and their stability analysis (Uniform stability, Asymptotic stability, Mittag-Leffler stability etc). We obtain various necessary and sufficient conditions for the existence, uniqueness and stability of the solutions to various delayed differential neural network models in terms of system parameters. The most significant contribution of this thesis is the introduction of a new discretization scheme to approximate the solutions of continuous-time fractional-order delayed neural networks via discrete difference equations, which has not been proposed so far to the best of our knowledge. We have also shown using exponential stability of the solutions that the obtained results are better approximation for the solution to the neural network models. Moreover, we have derived the global Mittag-Leffler stability of a complex valued fractional-order neural network with discrete and distributed time delay, which is quiet remarkable. Furthermore, the stability and synchronization has been investigated for various classes of neural networks, such as impulsive neural networks, projection neural networks, ring neural networks with reaction-diffusion terms. Various results have been derived to analyze the dynamic behaviour of the solutions to these models. The obtained results provide better understanding of the local as well as global behaviour of the various models. Numerical simulations have also been carried out to support the theoretical analysis. The main tools used in this study are fixed point methods, Lyapunov function, generalized Gronwall inequality, Razumikhin technique and fractional Adams-Bashforth-Moulton predictor-corrector numerical scheme.

Keywords: Neural Networks, Time delay, Fractional operators, Mittag-Leffler function, Fixed point methods, Lyapunov function, Gronwall inequality, Razumikhin technique, Fractional Adams-Bashforth-Moulton predictor-corrector numerical technique.

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Contents