



PHONON ASSISTED NON-LINEAR INTERACTION AND QUANTUM
ENTANGLEMENT IN SEMICONDUCTOR CAVITY QUANTUM ELECTRO
DYNAMICS

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of

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Kamand.

CERTIFICATE

This is to certify that the thesis titled “Phonon assisted non-linear interaction and quantum entanglement in semiconductor cavity quantum electro dynamics”, submitted by Harmanpreet Singh to the Indian Institute of Technology, Mandi, for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma.

Dr.Pradyumna Pathak

DECLARATION BY THE CANDIDATE

I hereby declare that the entire work embodied in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of Dr.P.K.Pathak that it has not been submitted elsewhere for any degree or diploma.

Harmanpreet Singh
Harmanpreet Singh

Dedicated to my family: my parents, my grandparents, my sister
my wife “Jaspreet” and daughter “Sargun”

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ABSTRACT

The work presented in this thesis evaluates the influence of phonon interaction on the solid-state quantum dots embedded inside microcavities. In the first chapter, we have introduced the basics of Cavity quantum electrodynamics (CQED). Quantum dots and microcavities are discussed in this chapter. Then polaron master equation is derived to study the influence of phonon interaction. Basics of quantum entanglement and its measures are also discussed in this chapter.

In the second chapter, the effect of exciton-phonon coupling on two-photon lasing in a single quantum dot embedded inside a photonic crystal cavity is discussed. Both incoherent and coherent pumping for achieving two-photon lasing is analyzed. The Wigner function is plotted to observe the squeezing in the cavity field. In the case of two-photon lasing, we do not find squeezing in the cavity field. However, we discuss the method of four-wave mixing for generating a continuous source of squeezed state using single QD.

In the third chapter, we have proposed the large phonon-assisted two-mode two-photon interaction in the system of two off-resonantly coupled QDs inside a bimodal photonic crystal cavity. We have observed that the cavity induced two-mode two-photon resonances, which appear for $g_1 \neq g_2$, is eliminated in the presence of exciton-phonon interactions and the phonon-assisted two-mode two-photon resonances occur at $\Delta_1 = \Delta_2$. It is observed that these interactions are more pronounced for positive detunings. In the fourth chapter, we have calculated the time-dependent concurrence for the two-photon NOON state and polarization-entangled state using a single biexcitonic quantum dot. In the last chapter, we have proposed an efficient two-photon source which uses an ultrashort Gaussian pulse for its operation.