

Almost Automorphic Solutions of Deterministic and Stochastic Dynamic Equations on Time Scales

A THESIS

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Dedicated

To

My Supervisor, My Parents, My Family, My Friends

And

My God



Declaration by the Guide

We hereby certify that the whole work in this thesis has been carried out by **Ms. Soniya (D16050)** under my supervision in the **School of Basic Sciences, Indian Institute of Technology Mandi** and no part of it has been submitted elsewhere for any degree or diploma.

Signature:

Name of the Guide: Dr. Syed Abbas

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Declaration by the Research Scholar

I, Soniya, Enrollment No. D16050, registered as a student in School of Basic Sciences, Indian Institute of Technology Mandi, India, under the supervision of **Dr. Syed Abbas**, do hereby submit my thesis report titled: **Almost Automorphic Solutions of Deterministic and Stochastic Dynamic Equations on Time Scales** in a printed as well as in an electronic version for holding in the library of record of the Institute.

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ABSTRACT

The present work deals with the study of the existence, uniqueness and stability of the almost automorphic solution and its generalization for deterministic and stochastic dynamic equations on time scales. We introduce the concept of equipotentially square-mean almost automorphic sequence, square-mean piecewise almost automorphic functions on time scales, time scale version of the Stepanov like square mean almost automorphic functions and some new results on composition theorem on time scales for the space of square-mean weighted Stepanov-like pseudo almost automorphic functions.

As applications, stochastic Cellular Neural Network, stochastic Nicholson's Blowflies model and non-autonomous Leslie-Gower prey-predator model are considered. First we derived some sufficient conditions for the existence of square mean almost automorphic solution for Stochastic Cellular Neural Network on time scales by using Krasnoselskii's fixed point theorem. Banach contraction principle is also used to show the uniqueness of solution with some conditions and exponential stability of the obtained unique solution is discussed by taken suitable Lyapunov function on time scales. Moreover, we have derived some sufficient conditions for the existence of square mean almost automorphic solution for stochastic Nicholson's Blowflies model on time scales. We establish appropriate conditions and results to discuss Ulam-Hyers-Rassias stability. Furthermore, the model with piecewise constant argument is considered. Then the approximate solution and a nicer bound of this model using discretization method is established. The significant property permanence is established along with the existence of almost automorphic solution of Leslie-Gower prey-predator model system. By constructing a suitable Lyapunov functional, presence of one of a kind all-around attractive positive almost automorphic solution of the system is obtained. Finally we discuss weighted pseudo almost automorphic solution of dynamic equation which models cellular neural network with time varying delay on time scales along the coefficients which are assumed to be Stepanov-like weighted pseudo almost automorphic functions. These obtained results improve and extend previous related work. Several examples with simulations for different-different time scales are explained for the adequacy of the hypothetical outcomes. Here the main techniques used are time scales theory, Banach fixed point principle, stochastic analysis theory, Krasnoselskii's fixed point theorem.

Keywords: *Time Scales, Almost Automorphy, Stepanov-like Almost Automorphy, Weighted Pseudo Almost Automorphy, Weighted Stepanov-like Pseudo Almost Automorphy, Stochastic Process, Evolution System, Cellular Neural network, Nicholson's Blowflies, Leslie-Gower Prey-Predator, Permanence, Ulam-Hyers-Rassias Stability, Exponential Stability, Approximation, Impulses, Existence, Banach Fixed Point Principle, Krasnoselskii's Fixed Point Theorem, Lyapunov Function.*

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