DEVELOPMENT AND ANALYSIS OF A CLASS OF TELEGRAPH-DIFFUSION MODELS: APPLICATION TO IMAGE RESTORATION

A Thesis

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of

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by

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(**D15028**)

Under the esteemed guidance of

Dr. Rajendra K. Ray

SCHOOL OF BASIC SCIENCES

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Dedicated
To My Beloved Parents
&
My Family
for their endless love, support, and compromise

CERTIFICATE BY SUPERVISOR

This is to certify that the thesis titled **"DEVELOPMENT AND ANALYSIS OF A CLASS OF TELEGRAPH-DIFFUSION MODELS: APPLICATION TO IMAGE RESTORATION"**, submitted by **SUDEB MAJEE (D15028)**, to the Indian Institute of Technology Mandi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma.

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DECLARATION BY RESEARCH SCHOLAR

I hereby declare that the entire work embodied in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of Dr. Rajendra K. Ray, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgments have been made wherever the work described is based on finding of other investigators.

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Signature:

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LIST OF PUBLICATIONS

The thesis is based on the following research papers published/submitted in international journals.

1. **S. Majee**, R. K. Ray, and A. K. Majee, A Gray Level Indicator-Based Regularized Telegraph Diffusion Model: Application to Image Despeckling, *SIAM Journal on Imaging Sciences (SIIMS)*, 13(2): 844-870 (2020).

2. **S. Majee**, S. K. Jain, R. K. Ray, and A. K. Majee, On the Development of a Coupled Nonlinear Telegraph-Diffusion Model for Image Restoration, *Computers & Mathematics with Applications (CAMWA)*, 80(7): 1745-1766 (2020).

3. S. K. Jain, **S. Majee**, R. K. Ray, and A. K. Majee, On the Existence and Uniqueness of Weak Solutions of a Coupled Diffusion System Related to Image Restoration (Under Review).

4. **S. Majee**, R. K. Ray, and A. K. Majee, An Effcient Nonlinear Hyperbolic-Parabolic Coupled PDE Model for Image Despeckling (Under Review).

5. **S. Majee**, S. K. Jain, R. K. Ray, and A. K. Majee, A Fuzzy Edge Detector Driven Telegraph Total Variation Model for Image Despeckling (Under Review).

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Abstract

In digital image processing applications, the detailed analysis of images always relies on the quality of the acquired image. In the real scenario, images are often degraded by different types of noises, e.g., additive, multiplicative, or mixed nature. These types of noises affect the acquired images during image formation, transmission, and recording processes and diminish the purity of the edge/texture information in the images. Hence the development of an advanced noise removal algorithm is always an essential aspect of the image processing community. It has been known for a long time that partial differential equation (PDE) based models can be efficient strategies for the noise removal process in digital images.

This thesis work mainly focuses on the development and analysis of a class of PDE based models and their applications to image restoration. Also, an extensive computational study is presented in this thesis to validate and confirm the ability of the developed models. In the existing literature, most of the researchers have concentrated their interest only on parabolic PDE based approaches for the noise removal process; a few researchers have used the hyperbolic-parabolic PDE based techniques only for the additive noise removal process. To the best of our knowledge, the present work marks the first step towards the use of hyperbolic-parabolic PDE based techniques for the multiplicative speckle noise elimination process. Details analysis of existence and uniqueness of weak solution is carried out for each developed model using fixed point theorem. Extensive numerical experiments are performed on natural as well as synthetic and real images to illustrate and compare the performance of each developed model with the performance of various existing approaches. The experimental results show that the proposed models are efficient for image denoising and can upgrade the visual appearance of the located edges better than other existing models.

Chapter 1 describes a concise introduction about the aims and objectives of the research problem considered here, along with details literature survey.

In **Chapter 2**, the existence and uniqueness of the weak solution of a coupled diffusion system is presented. Moreover, the computational experiments show that the considered model could be applied for image denoising.

A coupled telegraph equation based image denoising model for additive Gaussian noise removal is proposed in **Chapter 3**. First, we verify that the present system has a unique global weak solution using Banach's fixed point theorem. Then apply this approach over a set of grayscale images to illustrate the superiority of the proposed model over the recently developed telegraph diffusion-based techniques as well as the model discussed in Chapter 2.

In **Chapter 4**, a gray level indicator based nonlinear telegraph diffusion model is presented for image despeckling. The proposed model uses the benefit of the combined effect of the diffusion equation as well as the wave equation. In this method, the diffusion coefficient depends not only on the image gradient but also on the gray level of the image, which controls the diffusion process better than only gradient-based diffusion techniques. Moreover, we establish the well-posedness of the system using Schauder fixed point theorem. We show the superiority of the proposed model over three recently developed techniques on a set of gray level test images. Also, check the noise removal ability of the present model over some real

SAR images corrupted by speckle noise with different noise levels. To the best of our knowledge, this is the first work that utilizes a telegraph diffusion based model for image despeckling.

In **Chapter 5**, a hyperbolic-parabolic coupled system is presented for image despeckling. A separate equation is used to calculate the edge variable, which improves the quality of edge information in the despeckled images. A well-posedness result of the proposed system is established via Schauder fixed point theorem. A generalized weighted average finite-difference scheme, along with the Gauss-Seidel iterative technique, is used to solve the coupled system. Computational analyses are reported to show the effectiveness of the proposed model, with recently developed models, over a set of test images contaminated by speckle noise. Additionally, we check the noise removal capability of the present model over real SAR and Ultrasound images corrupted by speckle noise. Overall, our study confirms that the proposed approach is more effective than the other existing image despeckling models.

In **Chapter 6**, we discuss a telegraph total variation based model, with fuzzy edge detector, for image despeckling. A new attempt has been made to utilize the benefits of the total variation framework and fuzzy set theory. The proposed approach combines the telegraph equation and the fuzzy edge detector function, which is robust to noise as well as preserves the image structural details efficiently. Moreover, we establish the existence and uniqueness of a weak solution of a regularized version of the proposed system using Schauder fixed point theorem. With the proposed model, despeckling is carried out on natural and real SAR images. The experimental results of the proposed model are reported, which found better in terms of noise suppression and detail preservation as compared to various existing approaches. To the best of our knowledge, this is the first work that utilizes a telegraph total variation based model for image despeckling.

Chapter 7 describes a computational study on different color images when they are degraded by additive Gaussian noise, multiplicative speckle noise, and blurring effect.

Chapter 8 provides a general conclusion of the whole thesis work and future scope of the research work.

Keywords: Image restoration; Nonlinear diffusion; Telegraph equation; Total-Variation methods; Gray level indicator; Fuzzy edge detector; Texture preservation; Finite difference method; Numerical scheme; Well-posedness; Weak solution; Fixed point theorem.

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