

Study of Dynamic Equations on Time Scales with Applications in Control Theory

A THESIS

submitted by

VIPIN KUMAR

(D16024)

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KAMAND-175005, INDIA

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Dedicated

To

My parents and brother Pankaj

Declaration by the Research Advisor

I hereby certify that the entire work in this thesis has been carried out by **Mr. Vipin Kumar (Enrollment No: D16024)** under my supervision in the **School of Basic Sciences**, Indian Institute of Technology Mandi and that no part of it has been submitted elsewhere for any degree or diploma.

Signature:

Name of the Guide: Dr. Muslim

Date:

Declaration by the Research Scholar

I hereby declare that the entire work embodied in this thesis is the result of investigations carried out by me in the **School of Basic Sciences**, Indian Institute of Technology Mandi, under the supervision of **Dr. Muslim** and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgments have been made wherever the work described is based on the finding of other investigators.

Place: IIT Mandi

Date: **19-02-2021**

A handwritten signature in black ink that reads "Vipin Kumar". The signature is written in a cursive style and is positioned above a horizontal line.

Signature:

Vipin Kumar

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“They know enough who know how to learn”

– Henry Adams

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ABSTRACT

Keywords: *Time scales, Dynamic equations on time scales, Dynamic inclusions, Switched systems, Instantaneous impulses, Non-instantaneous impulses, Existence, Uniqueness, Stability, Hyer-Ulam stability, Controllability, Observability, Semigroup theory, Evolution operator theory, Banach fixed point theorem, Dhage fixed point theorem.*

The present research work deals with the investigation of various kinds of dynamic equations on time scales in finite as well as in infinite-dimensional spaces. This work provides insight into the different types of existence, uniqueness, stability, controllability and observability problems for dynamic equations with impulsive conditions on time scales. We establish the controllability results for time-varying neutral differential equations with impulses on time scales in a finite-dimensional space \mathbb{R}^n . Also, we study the controllability results for Volterra integro-dynamic inclusions with impulsive conditions on time scales. Further, we study the controllability and observability results for a dynamic system with non-instantaneous impulses on time scales in a finite-dimensional space \mathbb{R}^n by using the variation of parameter and Gramian matrices. Next, we establish existence, uniqueness, stability and controllability results for Volterra integro-dynamic systems with non-instantaneous impulses on time scales by introducing the Gramian type matrices. Furthermore, we establish some necessary and sufficient conditions of controllability for a class of impulsive switched systems with non-instantaneous jumps on time scales by using the parameter variation method and some Gramian matrices. Moreover, we give some conditions under which the time-invariant impulsive switched system is controllable. Finally, we discuss the controllability results for a class of abstract integro-hybrid evolution systems with impulses on time scales by using the semigroup and evolution operator theory. Several examples have been provided in order to make our theoretical analysis more concrete. In this work, the main techniques used are time scales theory, parameter variation method, Gramian matrices, multivalued map theory, fixed point theorems, semigroup and evolution operator theory.

Contents

Acknowledgement	i
Abstract	iii
List of Figures	vii
1 Introduction and preliminaries	1
1.1 Introduction	1
1.1.1 Description of Time Scales Theory	1
1.1.2 Basic Concept of Control Theory	4
1.1.3 Switched Systems	6
1.1.4 Some Special Kinds of Differential Equations	8
1.1.5 Abstract Formulation	11
1.2 Literature Survey	13
1.3 Preliminaries	21
1.3.1 Time Scales Theory	21
1.3.2 Semigroup Theory on Time Scales	29
1.3.3 Stability	30
1.3.4 Important Results	32
1.4 Research Objectives and Methodology	33
1.5 An Outline of Dissertation	37
2 Controllability of neutral differential equations with impulses on time scales	39
2.1 Introduction	39

2.2	Preliminaries	41
2.3	Controllability	43
2.4	Controllability of Integro-Differential Equation	47
2.5	Controllability of Nonlocal Problem	51
2.6	Example	54
2.7	Conclusion	55
3	Controllability results for Volterra integro-dynamic inclusions with impulses on time scales	57
3.1	Introduction	57
3.2	Preliminaries	59
3.3	Controllability	65
3.4	Controllability of Nonlocal Problem	69
3.5	Example	71
3.6	Conclusion	74
4	Controllability and observability for dynamic systems with non-instantaneous impulses on time scales	75
4.1	Introduction	75
4.2	Preliminaries	77
4.3	Controllability	77
4.4	Observability	86
4.5	Examples	90
4.6	Conclusion	101
5	Existence, stability and controllability results for Volterra integro-dynamic systems with non-instantaneous impulses on time scales	105
5.1	Introduction	105
5.2	Preliminaries	107
5.3	Existence and Stability	110
5.4	Controllability	115

5.4.1	Controllability of Linear Impulsive System	115
5.4.2	Controllability of Nonlinear Impulsive system	118
5.5	Example	124
5.6	Conclusion	128
6	Controllability results for a class of time-varying switched dynamic systems with non-instantaneous impulses on time scales	130
6.1	Introduction	130
6.2	Preliminaries	132
6.3	Main Results	135
6.4	Example	143
6.5	Conclusion	147
7	Results on abstract integro-hybrid evolution systems with impulses on time scales	150
7.1	Introduction	150
7.2	Preliminaries	152
7.3	Existence and Stability	154
7.4	Controllability	161
7.5	Examples	170
7.6	Conclusion	178
8	Conclusion and future scope	181
8.1	Conclusion	181
8.2	Future Scope	182
	List of Publications	184
	Bibliography	187