CONTINUUM MODELLING OF SNOW AVALANCHES USING AN ADAPTIVE-MESH OPEN-SOURCE FINITE ELEMENT FRAMEWORK

submitted by

NEERAJ KUMAR SINGH

for the award of the degree

of

MASTER OF SCIENCE

(by Research)

Supervisor: Dr GAURAV BHUTANI

SCHOOL OF ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY MANDI

JULY 2021

DECLARATION

This is to certify that the thesis titled "**Continuum modelling of snow avalanches using an adaptive-mesh open-source finite element framework**", submitted by me to the Indian Institute of Technology Mandi, for the award of the degree of Master of Science (by Research), is a bona fide record of the research work carried out by me under the supervision of **Dr Gaurav Bhutani**. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Place: IIT Mandi

Date: 16th July 2021

Neeraj Kumar Singh

THESIS CERTIFICATE

This is to certify that the thesis titled "**Continuum modelling of snow avalanches using an adaptive-mesh open-source finite element framework**", submitted by **Neeraj Kumar Singh** to the Indian Institute of Technology Mandi, for the award of the degree of Master of Science (by Research), is a bona fide record of the research work carried out by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr Gaurav Bhutani
Date: 16th July 2021

ACKNOWLEDGEMENTS

Firstly, I am extremely grateful to my supervisor, Dr Gaurav Bhutani for his invaluable advice, continuous support, and patience during my MS study. His immense knowledge and plentiful experience have encouraged me in all the time of my academic research and daily life.

I would also like to express my gratitude to all the APC members of Indian Institute of Technology Mandi–Dr P. Anil Kishan, Dr Pradeep Kumar, Dr Rajendra Kumar Ray, Dr Mousmi Mukherjee–for evaluating my research work yearly and providing comments and suggestions, which were valuable for this research work.

Additionally, I would like to thank Dr Arpan Gupta, for helping and supporting me in the early days of my academic journey. I thank all the members of the multiphase flow research lab (MFRL) and high performance computing (HPC) facility, IIT Mandi. Special thanks to Nikhil, Shreesh, Kamal and Manisha whose kind help and support that have made my study and life here a wonderful time. A vote of thanks to Arya Priyadarshini and Arpit Dwivedi for their support during my thesis writing. Finally, I express my gratitude to my parents and my siblings. Without their tremendous understanding and encouragement in the past few years, it would be impossible for me to complete my study.

ABSTRACT

A snow avalanche is a natural free-surface flow phenomenon that occurs in mountainous regions frequently. The mountainous areas of the Himalayas and the Alps have witnessed many casualties, including the collapse of buildings, roads, and highways. While designing appropriate preventive measures, bridges, and defense structures, a detailed analysis is crucial. The continuum conceptualisation of the flow results in mass and momentum partial differential equations, which can be solved to obtain the flow fields such as pressure, velocity and fluid interface. A constitutive (or rheological) model is required to "close" the above set of governing equations. Many past experiments reveal that flowing snow behaves similar to non-Newtonian fluids. The resultant coupled equations are non-linear partial differential equations, which require a numerical method to solve them.

The present work deals with the continuum modelling of snow avalanches, emphasising the non-Newtonian fluid dynamics. An implementation of non-Newtonian models was carried out in an open-source finite element method (FEM) based computational fluid dynamics (CFD) code. FEM provides added advantages of dealing with complex geometries using unstructured, anisotropic meshes. The accuracy of the solution increases by changing the order of polynomial in the basis functions in the discretisation. The credibility of the new implementation was ensured at each step through rigorous verification, benchmarking, and validation.

A channel flow problem was first solved with the power-law rheological model, and the results were verified with the existing analytical solutions. Different discretisation schemes were tested for the velocity-pressure element pair to demonstrate the order of convergence. Benchmarking was then performed for a 2-D lid-driven cavity problem (using the power-law fluid model) against the lattice Boltzmann method (LBM). A 2-D dam-break problem with the Carreau-Yasuda (CY) rheological model was then simulated and validated with an experiment. Another validation was carried out by simulating a snow-chute problem using the Cross rheological model, followed by a real snow avalanche simulation on actual mountain topography. Since the existing CFD solvers are compute intensive while dealing with three-dimensional domains, the novelty of the current work lies in modelling snow avalanche dynamics on real mountain topography using parallel, unstructured, anisotropic mesh adaptivity. The current framework can be extended to solve industrial problems like the flow of materials in process industries, pharmaceuticals, and mining.

TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS	i
	ii
ABSTRACT	iii
LIST OF TABLES	x
LIST OF FIGURES	xiv
NOMENCLATURE	xv
CHAPTER 1: INTRODUCTION	1
1.1 Motivation	1
1.2 Non-Newtonian fluids	2
1.3 Literature survey	3
1.4 Novelty	5
1.5 Objectives	6
1.6 Thesis outline	6
1.7 Publications	8
CHAPTER 2: MATHEMATICAL MODELING	9
2.1 Overview	9
2.2 Governing equations	9
2.3 Constitutive equations	11
2.3.1 Power-law model	11
2.3.2 Carreau-Yasuda model	11
2.3.3 Cross model	12
2.4 Multimaterial approach	12

Page

CHAPTER 3: NUMERICAL MODELLING 14

3.1 Overview 14

3.2 FEM formulation 15

3.2.1 Continuous Galerkin method 17

3.2.2 Discontinuous Galerkin method 19

3.3 Control volume discretisation 22

3.4 Time discretisation 23

3.5 Non-linear relaxation 24

3.6 Linear solvers 24

3.7 Mesh adaptivity 25

3.7.1 Metric estimation 26

3.7.2 Mesh generation 27

3.7.3 Field interpolation 27

3.8 Code parallelisation 28

3.9 Solution algorithm 28

3.10 Implementation of non-Newtonian model 29

3.11 Summary 31

CHAPTER 4: VERIFICATION: CHANNEL FLOW 32

4.1 Overview 32

4.2 Background 32

4.3 Channel flow problem setup 33

4.3.1 Geometry 33

4.3.2 Initial and boundary conditions 34

4.3.3 Physical and numerical parameters 34

4.4 Verification 35

4.5 Power-law index and accuracy 37

4.6 Order of convergence 38

4.7 Adaptive mesh simulation 39

4.8 Summary 40

CHAPTER 5: BENCHMARKING: COUETTE FLOW 41

5.1 Overview 41

5.2 Background 41

5.3 Problem setup 42

5.3.1 Geometry 42

5.3.2 Initial and boundary conditions 43

5.3.3 Physical and numerical parameters 43

5.4 Benchmarking 44

5.5 Reynolds number effect 46

5.6 Power-law index effect 49

5.7 Adaptive mesh simulation 50

5.8 Summary 51

CHAPTER 6: VALIDATION: DAM-BREAK PROBLEM 52

6.1 Overview 52

6.2 Background 52

6.3 Problem-setup 53

6.3.1 Geometry 53

6.3.2 Initial and boundary condition 54

6.3.3 Physical and numerical parameters 54

6.4 Validation 57

6.5 Impact force 59

6.6 Summary 59

CHAPTER 7: SNOW AVALANCHE DYNAMICS 61

7.1 Overview 61

7.2 Background 62

7.3 Snow chute flow 63

7.3.1 Geometry 63

7.3.2 Initial and boundary conditions 64

7.3.3 Physical and numerical parameters 65
7.3.4 Validation 66
7.4 Mountain avalanche simulation 69
7.4.1 Geometry 70
7.4.2 Initial and boundary conditions 71
7.4.3 Physical and numerical parameters 71
7.4.4 Snow avalanche simulation results 72
7.5 Summary 85

CHAPTER 8: CONCLUSION 86
8.1 Future work 87

APPENDIX A: Analytical solution of the channel flow problem for a power-law fluid 88

APPENDIX B: Comparison of 2-D and 3-D snow avalanche on Griffon peak with extended snow mass in the z-direction 90

REFERENCES 95