STUDY OF CHAOS AND PATTERN FORMATION ANALYSIS IN ECOLOGICAL SYSTEMS

A Thesis Submitted

in Accordance with the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

By

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Dedicated to Papa (Shri. Ramdhyan Singh)
&
Mummy (Shrimati. Subhawati Devi)
"For their love, endless support, encouragement & sacrifices"

THESIS CERTIFICATE

This is to certify that the work contained in the thesis entitled **"Study of Chaos and Pattern Formation Analysis in Ecological Systems"** being submitted by **Mr. Vikas Kumar (Roll No: D16025)** has been carried out under my supervision. In my opinion, the thesis has reached the standard fulfilling the requirement of regulation of the Ph.D. degree. The results embodied in this thesis have not been submitted elsewhere for the award of any degree or diploma.

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Declaration by the Research Scholar

I hereby declare that the entire work carried out in this thesis is the result of investigations carried out by me in the School of Basic Sciences, Indian Institute of Technology Mandi, under the supervision of **Dr. Nitu Kumari**, and that it has not been submitted elsewhere for any degree or diploma. In keeping with the general practice, due acknowledgements have been made wherever the work described is based on findings of other investigators.

Kamand, 175075

Date:

(Vikas Kumar)

या कुन्देन्दुतुषारहारधवला या शुभ्रवस्त्रावृताया
वीणावरदण्डमण्डितकरा या श्वेतपद्मासना।
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सा मां पातु सरस्वती भगवती निःशेषजाड्यापहा॥

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ABSTRACT

Most of the ecological systems are nonlinear in nature, and many of them are full of complexity and disorder. Mathematical modeling of these systems is an essential tool to understand the dynamical behavior of the system. Mathematical models are governed by nonlinear differential equations and exhibit various complex dynamics, including limit cycle, period-doubling, and chaos. Chaos in biological systems leads to serious imbalances and even collapse of ecosystems. This necessitates the study of the control of chaos. Chaos control methods stabilize unstable periodic orbits (including unstable equilibria) or reduce the leading positive Lyapunov exponent of the dynamical system. If chaos is controlled in a food chain system, then the species can change the system's irregular behaviour and bring the system's dynamics to an equilibrium state. Also, the obtained results will contribute to constructing an effective control policy to make the system permanent. Motivated by this, we have controlled chaotic dynamics in food chain models using various ecological factors, namely, fear effect, Allee effect, refugia, and cannibalism.

Spatial distribution of species is an interesting topic in population ecology, especially with community and landscape points of view. Different factors are responsible for the heterogeneous spatial distribution of population, for example, biotic factors such as predation and vegetation and abiotic factors such as rainfall, temperature and altitude. Reaction-diffusion systems represent these spatial processes along with the temporal evolution of species and lead to pattern formation phenomenon. Pattern formation is one of the typical features in ecosystems, which can characterize the relationship between population and space-time structure and monitor the functioning

of ecosystems. Hence, in order to understand the interaction between the temporal and spatial processes, and distribution of the species, we have spatially extended food chain systems and performed pattern formation analysis.

In this thesis, we have formulated deterministic mathematical models to study the impact of various ecological factors, namely, Allee effect, cannibalism, fear effect, group defense, and refugia, on the system dynamics and pattern formation. We have investigated two types of dynamical systems in population dynamics, namely, (i) Temporal system, modeled via the system of nonlinear ordinary differential equations (ODE) and (ii) Spatially extended system, modeled via the system of nonlinear parabolic partial differential equations (PDE). In the case of temporal systems, the prey-predator model and food chain models are analyzed with the aid of the theory of differential equations and dynamical system tools such as stability, bifurcation, persistence and permanence. Biologically relevant equilibrium points are obtained for each model system. Linearization method, Lyapunov function and center manifold theory are used to obtain the local stability of the system. The existence of Hopf bifurcation is proved, which occurs due to local birth and death of periodic oscillations. Moreover, its directions are measured using the normal form theory. Persistence and permanence of the system are shown to predict the long-term behavior of the system. Positivity and boundedness are also presented for the proposed dissipative systems. The nonlinear systems exhibit chaotic dynamics, which are successfully controlled using ecological factors. The spatiotemporal systems are also analyzed, where Turing instability and Hopf bifurcation are proved to ensure the existence of Turing and non-Turing patterns. Various types of patterns such as hot-spot, cold-spot, labyrinth and stripe patterns are obtained in one and two-dimensional spatial domains. Spatiotemporal chaos is

investigated in spatially extended systems. Numerical simulations are performed to understand the system dynamics and to demonstrate the impact of ecological factors on temporal and spatial interactions.

Keywords: Dynamical system, Fear effect, Allee effect, Refugia, Group defense, Cannibalism, Stable focus, Limit cycle, Period-doubling, Chaos, Local stability, Hopf bifurcation, Persistence, Permanence, Turing instability, Turing patterns, Non-Turing patterns, Spatiotemporal chaos, Numerical simulation

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